Jean-Francois Cossette (a graduate-student visitor from the Department of Physics at the University of Montreal) continued his study of improved trajectory schemes for semi-Lagrangian integrations of fluid equations. In contrast to Eulerian finite-volume methods — which typically ensure that velocities transporting dependent fluid variables satisfy the mass continuity equation — semi-Lagrangian models commonly disregard the mass continuity constraint in the evaluation of the departure points of flow trajectories. Jean-Francois evolved an original scheme that improves on this aspect of semi-Lagrangian models. The key element of the approach is solving a Monge-Ampere nonlinear elliptic equation. To prove the concept, Jean-Francois developed an effective Monge-Ampere solver for incompressible fluids, where the continuity constraint $\text{div} \ V = 0$ amounts to the flow Jacobian equal unity. A series of numerical experiments ranging from elementary passive scalar advection (see figure 1) to 3D turbulent flows indicates that the approach may benefit the conservativity and overall accuracy of semi-Lagrangian models. Jean-Francois presented preliminary results of his research at the NCAR Summer School on Geophysical Turbulence, 14 July - 1 August 2008, Boulder, Colorado; http://www.image.ucar.edu/Workshops/TOY2008/focus4/lectures.shtml. This long-term interdisciplinary project is a part of a cooperative graduate thesis, with Paul Charbonneau (University of Montreal) and Piotr Smolarkiewicz (NCAR) as co-directors.
Figure 1: Semi-Lagrangian advection of a passive scalar in a vortical flow. The left and right plates show the results, respectively, with and without trajectory correction via Monge-Ampere solver. The three curves displayed in each panel are for the exact result and numerical solutions after 16 and 31 rotations of the tracer, in the order of decreasing height (h) respectively.