1. Wavelet estimation of coherent modes in turbulence;

2. Development of spectral-element and finite-volume methods;

3. Other research

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1 **Historical & scientific context and strategic alignment**

Wavelet analysis is now a method of well recognized effectiveness in a range of disciplines, including atmospheric science e.g., 39 publications from NCAR from [10] to [11]. Our recent research includes applying a wavelet-based criterion to identify coherent structures in turbulence simulations, §2.1. Also, as detailed in recent reports [6 §1; 7; 8 & op. cit. therein], a need is well known and pursued at NCAR and elsewhere, to create high-resolution numerical simulation methods for atmospheric dynamics, that are accurate similarly to spectral methods and efficient similarly to finite-element type methods. A research code that demonstrates the strengths of the spectral-element method (SEM) with dynamic-nonconforming adaptive mesh refinement (AMR) is the geophysical-astrophysical spectral-element adaptive refinement (GASpAR) code (§§2.2, 2.3) [18, 19]. Especially for long-time, potentially parameter-sensitive simulations such as climate, spatial discretizations should accurately validate physical conservation laws such as mass and energy; this motivates another part of our research (§§2.3, 2.4).

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2 2009 accomplishments

2.1 Wavelet analysis of rotating turbulence

At least 10 years ago M. Farge and collaborators proposed an algorithm for identification of coherent structures in turbulence [3], that has since been applied to a wide variety of turbulence simulations by those [e.g., 16] and other authors [e.g., 2]. In principle the algorithm depends only on the variance-estimate $\sigma^2$ and grid-size $N$ of the analyzed data, although the variance estimation method, convergence criteria etc. are adjustable parameters in a sense. In summary, the algorithm identifies “coherent” vorticity structures $\omega_c \equiv \omega - \omega_i$ as the residual of removing “incoherent” structures from the total $\omega \equiv \nabla \times \vec{u}$, and the $\omega_i = \sum_{j \in J_{\text{final}}} \omega_j W_j$ as the sum over a set of wavelet-coefficients $\omega_j \equiv \langle \omega \mid W_j \rangle$ that has converged after an iterative procedure to remove components with magnitude exceeding an adaptive threshold $\tau_{\text{new}} = \sqrt{(2 \ln N)N^{-1} \sum_{j \in J_{\text{final}}} |\omega_j|^2}$, where $J_{\text{old}} = \{ j \in J_{\text{initial}} : |\omega_j| \leq \tau_{\text{old}} \}$. We are currently applying this algorithm to the largest rotating-turbulence simulation ever performed, a size-$1536^3$ pseudospectral computation with Reynolds number $Re \approx 5600$ and Rossby number $Ro \approx 0.06$ [14]. For comparison, we similarly analyzed a 1024$^3$ computation with $Re \approx 6200$ and $Ro = \infty$ [13, run VII]. We find from the histograms and sample-moments of the total velocity that:

- considering the mean $\mu_1 \equiv \langle v_s \rangle$ of all 3 components $s = x, y, z$ of $\vec{v} = \vec{u}$ and $\vec{v} = \vec{\omega}$, the incoherent mean $\mu_{1,i}$ is closer to the total mean $\mu_1$ than is the coherent mean $\mu_{1,c}$;

- considering the variance $\mu_2 \equiv \langle (v_s - \mu_1)^2 \rangle$, skewness $\mu_3 \equiv \mu_2^{-3/2}\langle (v_s - \mu_1)^3 \rangle$ and excess kurtosis $\mu_4 \equiv \mu_2^{-2}\langle (v_s - \mu_1)^4 \rangle - 3$, for each $v_s$ and $m > 1$, $\mu_{m,c}$ is much closer to $\mu_m$ than is $\mu_{m,i}$;

- the histogram of $v_{s,c}$ is much closer to that of $v_s$ than is that of $v_{s,i}$;

- the coherent parts $v_{s,c}$ comprise only 2% of the coefficients $v_{s,j}$ but represent at least 90% of the energy and enstrophy, in that $\mu_2 \approx \mu_{2,c} \gg \mu_{2,i}$;

- the incoherent parts $v_{s,i}$ are more Gaussian in the sense that $\mu_{3,i} \ll \mu_3$ and (for $\vec{v} \neq \vec{u}$) $\mu_{4,i} \ll \mu_4$; and

- the incoherent parts $u_{s,i}$ are less Gaussian in the sense that $\mu_{4,i} \gg \mu_4$.

We believe that the last point is probably due to the large-scale deterministic Arn’old-Childress-Beltrami forcing of this run, that seems to broaden the peak and thin the tails of the $u_s$ histograms and reduce $\mu_4 < 0$. From a preliminary literature review it seems that Farge et al. always analyzed stochastically forced simulations.

2.2 GASpAR application to decaying 2D vortex interactions at high Reynolds number

The work presented earlier [7, 8, & op. cit. therein] has been published [9]. For a case characterized by strong nonlinear interactions between vortex structures similar to those in the atmosphere, dynamic-AMR GASpAR simulations illustrated the accuracy of GASpAR compared to several well-resolved standard spectral simulations in the literature. In revising the paper we discovered that the energy-spectrum power-law exponent varies over time to pass through its trough just when the enstrophy-dissipation rate (pseudointrophy/enstrophy ratio) passes through its crest; after this time, temporal self-similarity could be expected.
2.3 Compatibility based exact semi-discrete local conservation properties for continuous SEM

As suggested by earlier studies [7, 8, & op. cit. therein], in ongoing collaboration with M. Taylor, we derived a formulation of SEM that is compatible on fully unstructured 3D grids. A recently published extension of this work with P.H. Lauritzen and A. St-Cyr is to derive a new formulation of the SEM advection operator that is conservative and non-oscillatory [20, 21, 22, 23]. An aspect of this formulation was tested in GASpAR [17]. Figure 1 shows the energy \( E \equiv \langle \vec{u} \cdot \vec{u} \rangle / 2 \) and energy-dissipation imbalance \( \Xi \equiv 1 + (dE/dt)/2Re^{-1}\Omega \) (where \( \Omega \equiv \langle \vec{\omega} \cdot \vec{\omega} \rangle / 2 \) is the enstrophy) for a GASpAR simulation of the unforced 2D Navier-Stokes eqs. using 64\(^2\) elements. For the conservative advection formula \( \vec{\omega} \times \vec{u} \) the expected value \( \Xi = 0 \) is more closely approached until \( t > 0.17t_{\text{eddy}} \), and less energy is dissipated until \( t > 0.19t_{\text{eddy}} \), than for the formula \( (\vec{u} \cdot \nabla)\vec{u} \). This simulation uses low polynomial degree \( p = 3 \) in each element and is under-resolved. At \( p = 8 \) it is resolved but the conservation improvement is not appreciable. Unfortunately the improved \( E \) conservation seems to come at the cost of eventual unbounded growth of \( \Omega \). Possibly an explanation could be inferred from the paper ref. 25, that states that this \( E \)-conservative formula is linearly unstable. Thus remedies may include using an unconditionally stable time marching scheme, or a time marching scheme that adapts the time step to the most unstable eigenvalue (so each step is always stable). It may be that the instability is due to the grid staggering, and could be remedied on a collocated grid.

2.4 A polyhedral finite-volume model of the global atmosphere

We performed analyses of conforming AMR methods that improve unstructured finite-volume simulation of shallow-water and similar flows in spherical and more general geometries [8]. This author contributed to ongoing work with the new FV model AtmosFOAM, that compared the accuracy and cost effectiveness of
four quasi-uniform meshes of the sphere: a cubed-sphere, reduced latitude-longitude, hexagonal-icosahedral and triangular-icosahedral [24].

2.5 Stochastic parameterization of spectral backscatter for NWP and climate model-error

This author supported the reformulation of a spherical-geometry stochastic-backscatter scheme [1] for the limited-area WRF model using non-periodic Fourier analysis. Our initial results include that the Brier skill score for zonal-wind forecasts at 12h, 36h and 60h are improved at all isobaric levels [17]. Much larger runs are pending, to prove the statistical significance of these improvements.

2.6 Lyapunov exponents of a moist low-order general circulation model

This author was approached by S. Vannitsem who sought my expertise [4, 5, & 5 other manuscripts] in regard to the moist low-order general circulation model of Lorenz [12]. That collaborator used my codes and other resources to conduct several studies of the effect of model parameters (e.g., ocean mixed-layer depth, water-vapor coupling) on predictability. We have been discussing intermediate results and exploring how to proceed.

3 2010 Plans

The appointment of this author concluded 2009 June 6 only due to "lack of available funding" [15]. I have greatly enjoyed my collaborations and friendships in the TNT and GTP and am looking forward to new projects in a new position in ESSL/MMM.

References


