Modeling of turbulence with rotation or magnetic fields

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Context  Since direct numerical simulations (DNS) of geophysical flows at realistic (high) Reynolds number $Re$ exceed technological limits, and since simple truncation may remove important physics, some sort of closure is required to model the effects of the unresolvable small scales on the numerically resolved scales. Large Eddy Simulations (LES) are widely used but the Reynolds number is not known; instead, one attempts to model the behavior of the flow in the limit of large $Re$. As Kolmogorov assumption of strict self-similarity in turbulence is known to break down, the magnitude of $Re$ can be important. Newer approaches model the effects of turbulence at higher $Re$ than are possible with a DNS on a given grid, by a combination of mimicking the effects the neglected scales may have on the resolved ones through the implementation of transport coefficients, of filtering of the small scales or more, as delineated below.

Modeling of rotating flows  The conceptual modeling framework developed recently ([1]; [2]) has been generalized to rotating flows using an isotropic spectral closure; indeed, one can show that isotropy recovers in the small scales that are modeled by the Large Eddy Simulation (LES) procedure, at least for moderate rotation rates. Both non-helical and helical forcing terms have been implemented, our LES allowing for taking into account the contribution of helicity to transport coefficients, helicity measuring a lack of mirror symmetry in the flow. We can recover DNS results at substantially lower costs; in the helical case, a DNS run on a grid of $1536^3$ points (world record), using a large allocation in the context of NCAR Accelerated Scientific Discovery, can be adequately modeled using grids of $96^3$ points ([3]; [4]), with a $\sim 10\%$ error on the energy at a fixed Reynolds number. Fig. 1 (left) shows the temporal evolution of the energy for the helical DNS, two LES and an under-resolved DNS. LES clearly works better than the latter, and one can thus pursue such substantive runs to longer times using the spectral LES. The flow develops strong laminar columnar vortices (Fig. 1, left center) that are fully helical. This model is presently being used to explore parameter space, e.g. in terms of Rossby and Reynolds numbers.

Euler dynamics  We have also pursued our search for equilibrium solutions to the ideal equations in the presence of helicity, as precursors of dissipative dynamics ([6]). Transient energy and helicity cascades leading to Kraichnan helical absolute equilibrium at small scales, are obtained for the first time. Strong helicity effects are found when using initial data concentrated at high wavenumbers. Using now low-wavenumbers initial conditions, similarities between the turbulent transient evolution of the ideal (time-reversible) system and viscous helical flows are found; the excess of relative helicity observed at small scales in the viscous run is related to the thermalization of the ideal flow (see Fig. 1, right center). The differences in the behavior of truncated Euler and Navier-Stokes are qualitatively understood using the concept of eddy viscosity using Markovianized models. The large scales of the truncated Euler equations are then shown to follow quantitatively an effective Navier-Stokes dynamics based on a variable (scale-dependent) eddy viscosity. We are now extending this work to MHD turbulence, both in two and in three dimensions.

Modeling of MHD turbulence  Once properly tested, models can allow an exploration of the dynamics of high Reynolds number flows. This is what has been pursued here, by (i) running for substantially longer
Figure 1: Left: Comparison of the temporal evolution of the energy for a DNS (solid, short line) and several models for a rotating flow, Rossby number $Ro \approx 0.06$, Reynolds number $Re \approx 5600$; the full DNS is run on a grid of $1536^3$ points; the under-resolved DNS with $160^3$ points (++) and the Chollet-Lesieur model (CL, grey line) are computed until $t \approx 30$, whereas the spectral LESPH (dash-dot) developed in (2) is pursued for twice that time; the CL and LESPH models are run on grids of $96^3$ points, representing a savings in CPU of almost $10^4$. Left center: Vertical velocity of the rotating flow run with LESPH, showing columnar structures in the direction of rotation, and in red, particle trajectories (imaging using VAPOR). Right center: Ratio of helicity to energy spectra $H(k)/E(k)$ of truncated Euler (blue dots) and Navier-Stokes (green $xx$), in the latter case at maximum energy-dissipation time; dash lines: $k^2$ and $k^{1/2}$, the former corresponding to the statistical equilibrium solution of Kraichnan at small scale; note the proportionality of $H(k)$ and $E(k)$ at large scale. Right: Energy spectra compensated by an Iroshnikov-Kraichnan law $k^{-3/2}$ (dots: kinetic, dash: magnetic, solid: total; dot-dash: Kolmogorov $k^{-5/3}$-compensated) using a Lagrangian model for MHD on a $3096^3$ equivalent grid run, unattainable today with a DNS.

This work supports the strategic priorities of NCAR by developing and testing multi-scale models of turbulence against massive DNS, and going further with the models than what is feasible today using DNS, in preparation of petascale.

References